Motion in Two Dimensions

Vector Properties and Operations
Motion under constant acceleration
Projectile Motion
Announcements

• Homework: 34 of you have signed up (out of 37)
  – Very good!!!

• e-mail distribution list: 16 of you have subscribed so far.
  – This is the primary communication tool. So subscribe to it ASAP.
  – A test message has been sent last Wednesday for verification purpose
  – There will be negative extra credit from this week
    • -1 point if not done by 5pm, Friday, Sept. 12
    • -3 points if not done by 5pm, Friday, Sept. 19
    • -5 points if not done by 5pm, Friday, Sept. 26

• Quiz #1:
  – Average score of the class: 3.2
  – Quizzes are 15% of the final grades
Displacement, Velocity and Speed

**Displacement**

\[ \Delta x = x_f - x_i \]

**Average velocity**

\[ v_x = \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t} \]

**Average speed**

\[ v = \frac{\text{Total Distance Traveled}}{\text{Total Time Spent}} \]

**Instantaneous velocity**

\[ v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \]

**Instantaneous speed**

\[ |v_x| = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \left| \frac{dx}{dt} \right| \]
Kinetic Equations of Motion on a Straight Line Under Constant Acceleration

\[ v_{xf}(t) = v_{xi} + axt \]  

Velocity as a function of time

\[ x_f - x_i = \frac{1}{2} v_x t = \frac{1}{2} (v_{xf} + v_{xi})t \]  

Displacement as a function of velocity and time

\[ x_f = x_i + v_{xi}t + \frac{1}{2} axt^2 \]  

Displacement as a function of time, velocity, and acceleration

\[ v_{xf}^2 = v_{xi}^2 + 2ax(x_f - x_i) \]  

Velocity as a function of Displacement and acceleration

You may use different forms of Kinetic equations, depending on the information given to you for specific physical problems!!
Coordinate Systems

- Makes it easy to express locations or positions
- Two commonly used systems, depending on convenience
  - Cartesian (Rectangular) Coordinate System
    - Coordinates are expressed in (x,y)
  - Polar Coordinate System
    - Coordinates are expressed in (r,θ)
- Vectors become a lot easier to express and compute

\[ (x_1, y_1) = (r \cos θ, r \sin θ) \]

\[ r = \sqrt{x_1^2 + y_1^2} \]

\[ \tan θ = \frac{y_1}{x_1} \]
Example

Cartesian Coordinate of a point in the xy plane are \((x,y)= (-3.50,-2.50)\) m. Find the polar coordinates of this point.

\[
\begin{align*}
\theta &= 180 + \theta_s \\
\tan \theta_s &= \frac{-2.50}{-3.50} = \frac{5}{7} \\
\theta_s &= \tan^{-1}\left(\frac{5}{7}\right) = 35.5^\circ \\
\therefore \theta &= 180 + \theta_s = 180^\circ + 35.5^\circ = 216^\circ
\end{align*}
\]
Vector and Scalar

Vector quantities have both magnitude (size) and direction

- **Force**, gravitational pull, momentum

- Normally denoted in **BOLD** letters, \( \mathbf{F} \), or a letter with arrow on top \( \vec{F} \)

- Their sizes or magnitudes are denoted with normal letters, \( F \), or absolute values: \( |\vec{F}| \) or \( |F| \)

Scalar quantities have magnitude only

- Energy, heat, mass, speed

- Can be completely specified with a value and its unit

- Normally denoted in normal letters, \( E \)

**Both have units!!!**
Properties of Vectors

- Two vectors are the same if their sizes and the directions are the same, no matter where they are on a coordinate system.

Which ones are the same vectors?

A=B=E=D

Why aren't the others?

C: The same magnitude but opposite direction: C=-A

F: The same direction but different magnitude
Vector Operations

• Addition:
  – Triangular Method: One can add vectors by connecting the head of one vector to the tail of the other (head-to-tail)
  – Parallelogram method: Connect the tails of the two vectors and extend
  – Addition is commutative: Changing order of operation does not affect the results
    \[ A + B = B + A, \quad A + B + C + D + E = E + C + A + B + D \]

\[
\begin{align*}
A + B & \quad B = \quad B \\
A & \quad A + B \\
\text{OR} & \quad \text{OR} \\
A & \quad A + B
\end{align*}
\]

• Subtraction:
  – The same as adding a negative vector: \[ A - B = A + (-B) \]

\[
\begin{align*}
A - B & \quad -B \\
A & \quad A - B
\end{align*}
\]

Since subtraction is the equivalent to adding a negative vector, subtraction is also commutative!!!

• Multiplication by a scalar is increasing the magnitude: \[ A, \quad B = 2A \]
Example

A car travels 20.0 km due north followed by 35.0 km in a direction 60.0° west of north. Find the magnitude and direction of resultant displacement.

\[ r = \sqrt{(A + B \cos \theta)^2 + (B \sin \theta)^2} \]
\[ = \sqrt{A^2 + B^2 (\cos^2 \theta + \sin^2 \theta) + 2AB \cos \theta} \]
\[ = \sqrt{A^2 + B^2 + 2AB \cos \theta} \]
\[ = \sqrt{(20.0)^2 + (35.0)^2 + 2 \times 20.0 \times 35.0 \cos 60} \]
\[ = \sqrt{2325} = 48.2 \text{ (km)} \]

\[ \theta = \tan^{-1} \frac{|B| \sin 60}{|A| + |B| \cos 60} \]
\[ = \tan^{-1} \frac{35.0 \sin 60}{20.0 + 35.0 \cos 60} \]
\[ = \tan^{-1} \frac{30.3}{37.5} = 38.9° \text{ to W wrt N} \]