Mechanics - Basic Physical Concepts

Mathematics

Quadratic Eq.: \( ax^2 + bx + c = 0 \), \( x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \)

Cartesian and polar coordinates:
\[ x = r \cos \theta, \quad y = r \sin \theta, \quad r^2 = x^2 + y^2, \quad \tan \theta = \frac{y}{x} \]

Trigonometry:
\[ \cos \alpha \cos \beta + \sin \alpha \sin \beta = \cos(\alpha - \beta) \]
\[ \sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2} \]
\[ \cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2} \]
\[ \sin 2 \theta = 2 \sin \theta \cos \theta, \quad \cos 2 \theta = \cos^2 \theta - \sin^2 \theta \]
\[ 1 - \cos 2 \theta = 2 \sin^2 \frac{\theta}{2}, \quad 1 + \cos 2 \theta = 2 \cos^2 \frac{\theta}{2} \]

Vector algebra:
\[ \vec{\mathbf{A}} \times \vec{\mathbf{B}} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \]

Resultant:
\[ \vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = (A_x + B_x, A_y + B_y) \]

Dot:
\[ \vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = A_x B_x + A_y B_y + A_z B_z \]

Cross product:
\[ \vec{\mathbf{C}} = \vec{\mathbf{A}} \times \vec{\mathbf{B}} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \]

C = \( \vec{\mathbf{A}} \times \vec{\mathbf{B}} \)

Calculating:
\[ \frac{dx}{dt} = x \frac{dx}{dt}, \quad \frac{d}{dt} \sin \theta = \cos \theta \]
\[ \frac{d}{dt} \cos \theta = -\sin \theta \]
\[ \frac{d}{dt} \text{const} = 0 \]

Measurements

Dimensional analysis, e.g.
\[ F = ma \rightarrow [M][L][T]^{-2} \]
\[ W = Fd \rightarrow [M][L][T]^{-2} \]

Summation:
\[ \sum_{i=1}^{N} (a_i x_i + b) = a \sum_{i=1}^{N} x_i + b N \]

Motion

One dimensional motion:
\[ v(t) = \frac{dx}{dt}, \quad a(t) = \frac{dv}{dt} \]

Average values:
\[ v = \frac{v_f + v_i}{2} \]

One dimensional motion (constant acceleration):
\[ v(t) = v_0 + at \]
\[ s(t) = \frac{1}{2} at^2 + v_0 t + s_0 \]

Nonuniform acceleration:
\[ x = x_0 + v_0 t + \frac{1}{2} a t^2 + \frac{1}{3} t^3 \]  
\[ \frac{d}{dx} F = ma \]
\[ p = \frac{m \omega}{2} \]

Projectile motion:
\[ t_{\text{fall}} = \frac{u}{g} \]
\[ h = \frac{1}{2} g t_f^2 \]

Circular:
\[ a_c = \frac{v^2}{r} \]
\[ v = \frac{2 \pi r}{T} \]
\[ f = \frac{1}{T} \]

Curvilinear motion:
\[ \ddot{v} = \dddot{v} + \ddot{a} \]

Force
\[ F = ma, \quad F \vec{\mathbf{q}} = \vec{\mathbf{F}} \]

Friction:
\[ F_{\text{static}} \leq \mu_s N \]

Equilibrium (concurrent forces):
\[ \sum \vec{F} = 0 \]

Energy

Work (for all F):
\[ W = F \Delta \vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = W_B - W_A = F s \]  
\[ W = F \cos \theta = F \cos \gamma = \sum \vec{F} \cdot d \vec{\mathbf{x}} \]  
\[ (\text{in Joules}) \]

Effects due to work done:
\[ F_{\text{ext}} = m \ddot{a} + F_c + f_{\text{nc}} \]

Kinetic energy:
\[ K = \frac{1}{2} m v^2 \]

Torque:
\[ \tau \vec{\mathbf{r}} = \vec{\mathbf{I}} \vec{\mathbf{F}} \]

Two-body collision:
\[ v_i^2 = v_i - v_{\text{cm}} \]

Elastic:
\[ v_i^2 = v_{\text{cm}} - v_{\text{int}} \]

Many body center of mass:
\[ v_{\text{cm}} = \sum m_i v_i \]

Rotation of Rigid-Body

Kinematics:
\[ \theta = \frac{x}{r}, \quad \omega = \frac{d\theta}{dt}, \quad \alpha = \frac{d\omega}{dt} \]

Moment of inertia:
\[ I = \sum m_i r_i^2 \]

Angular momentum:
\[ \vec{\mathbf{L}} \]

Torque:
\[ \tau = i \vec{\mathbf{r}} \vec{\mathbf{F}} \]

Rolling, angular momentum and torque:
\[ \vec{\mathbf{R}} = \vec{\mathbf{F}} \]

Angular momentum:
\[ \vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} \]

Gyroscopic:
\[ \omega \vec{\mathbf{G}} = \frac{d\vec{\mathbf{L}}}{dt} \]

Static equilibrium:
\[ \sum \vec{F} = 0 \]

Elastic modulus:
\[ \frac{F}{A} \]

Strain:
\[ \Delta L/L = \varepsilon \quad \Delta x/x = \varepsilon \quad \Delta V/V = 0 \]
Oscillation motion
\[ f = \frac{1}{T}, \quad \omega = \frac{2\pi}{T} \]

**S H M:**
- \( v = v_{\text{max}} \sin(\omega t + \delta) \)
- \( x = x_{\text{max}} \cos(\omega t + \delta) \)
- \( v = -v_{\text{max}} \sin(\omega t + \delta) \)
- \( x_{\text{max}} = A \)
- \( v_{\text{max}} = \omega A \)
- \( a = -a_{\text{max}} \cos(\omega t + \delta) = -\omega^2 x \)
- \( a_{\text{max}} = \omega^2 A \)

\[ E = K + U = K_{\text{max}} = \frac{1}{2} m (\omega A)^2 = U_{\text{max}} = \frac{1}{2} k A^2 \]

*Spring:* \( m a = -k x \)

Simple pendulum: \( m a = m a \ell = -m g \sin \theta \)

**Physical pendulum:** \( \tau = I \alpha = -m g d \sin \theta \)

**Torsion pendulum:** \( \tau = I \alpha = -k \theta \)

**Gravity**
\[ F_{G} = -G \frac{m_{1} m_{2}}{r_{12}^{2}}, \quad \text{for} \ r \geq R, \quad g(r) = G \frac{M}{r^{2}} \]

- \( G = 6.67259 \times 10^{-11} \text{Nm}^{2}/\text{kg}^{2} \)
- \( R_{\text{earth}} = 6370 \text{ km}, \quad M_{\text{earth}} = 5.98 \times 10^{24} \text{ kg} \)

**Circuml orbit:**
\( a = \frac{r}{2} = \omega^{2} r = \left( \frac{2\pi}{T} \right)^{2} r = g(r) \)

\[ U = -G \frac{m_{1} M}{r}, \quad E = U + K = G \frac{m M}{2 r} \]

\[ F = -\frac{d U}{d r} = -m G \frac{M}{r^2} = -m \frac{2 \pi^{2}}{T^{2}} \]

**Kepler's Laws of planetary motion:**
- **i)** elliptical orbit, \( r = \frac{b^{2}}{a^{2}} \sin \theta, \quad r_{1} = \frac{b^{3}}{a^{3}} \sin \theta, \quad r_{2} = \frac{b^{3}}{a^{3}} \)
- **ii)** \( L = r m \frac{\Delta \theta}{2 \pi} = \frac{b^{3}}{2 \pi} \), \( a = \frac{b^{3}}{2 \pi} \), \( T = \frac{b^{3}}{2 \pi} \)
- **iii)** \( G \frac{m_{1} M}{r^{2}} = \left( \frac{2 \pi}{T} \right)^{2} r \), \( a \Delta \theta = \frac{b^{3}}{2 \pi} \)

**Escape kinetic energy:** \( E = K + U(R) = 0 \)

**Fluid mechanics**
- **Pascal:** \( P = \frac{F}{A} = \frac{F}{A} \)
- **Archimedes:** \( B = Mg \), \( \text{Pascal}=\text{N/m}^{2} \)

\[ P = \left( \frac{M}{\rho} \right) V, \quad V = \rho \frac{M}{\rho} \]

\[ F = \int P dA = \rho g \ell \Delta h \]

**Continuity equation:** \( A v = \text{constant} \)

**Bernoulli:** \( P + \frac{1}{2} \rho v^{2} + \rho g \ell = \text{const} \), \( P \geq 0 \)

**Wave motion**
- **Traveling waves:** \( y = f(x - vt), \quad y = f(x + vt) \)
- **Oscillatory z direction:** \( y = \sin(\omega x - vt - \phi) \)

\[ T = \frac{1}{f}, \quad \omega = \frac{2\pi}{T}, \quad k = \frac{2\pi}{\lambda}, \quad v = \frac{\lambda}{T} \]

**Along a string:** \( v = \sqrt{\frac{T}{\mu}} \)

**General:** \( \Delta E = \Delta K + \Delta U = \Delta K_{\text{max}} \)

\[ P = \frac{\Delta E}{\Delta t} = \frac{1}{2} m \frac{\Delta v}{\Delta t} (\omega A)^2 \]

**Waves:**
\[ \frac{\Delta E}{\Delta t} = \frac{\Delta M}{\Delta t} \frac{\Delta A}{\Delta t} \cdot v \]

\[ P = \frac{1}{2} \mu v (\omega A)^2, \quad \mu = \frac{\Delta M}{\Delta t} \]

**Circuml:**
\[ \frac{\Delta A}{\Delta t} = \frac{\Delta A}{\Delta t} \cdot \frac{\Delta A}{\Delta t} = 2 \pi r v \]

**Spherical:**
\[ \frac{\Delta v}{\Delta t} = \frac{4 \pi r^{2} v}{\Delta t} \]

**Sound**
\[ v = \sqrt{\frac{T}{\rho}}, \quad s = s_{\text{max}} \cos(\omega x - vt - \phi) \]

\[ \Delta P = -B \frac{\Delta v}{\Delta t} = -B \frac{\Delta x}{\Delta t}, \quad \Delta P_{\text{max}} = B k s_{\text{max}} = \rho v w s_{\text{max}} \]

**Piston:**
\[ \Delta m = \frac{\Delta v}{\Delta t} = \frac{A \Delta x}{\Delta t} = \rho A v \]

**Intensity:**
\[ I = \frac{p}{\frac{V}{A}} = \frac{1}{2} \rho v (\omega s_{\text{max}})^{2} \]

**Intensity level:** \( \beta = 10 \log_{10} \frac{I_{0}}{I_{0}} = 10^{-12} \text{W/m}^{2} \)

**Plane waves:** \( \psi(x,t) = c \sin(k x - vt + \phi) \)

**Circular waves:**
\[ \psi(r,t) = \frac{1}{r} \sin(k r - vt + \phi) \]

**Spherical:**
\[ \psi(r,t) = \frac{1}{r} \sin(k r - vt + \phi) \]

**Doppler effect:**
\[ \lambda = \frac{\nu}{\nu_{0}}, \quad f_{0} = \frac{1}{\nu}, \quad f' = \frac{\nu'}{\nu_{0}} \]

**Temperature and heat**
- **Conversions:** \( F = \frac{9}{5} C + 32 \), \( K = C + 273.15 \)
- **Constant volume gas thermometr:** \( T = a P + b \)
- **Thermal expansion:** \( \alpha = \frac{\Delta l}{l}, \quad \beta = \frac{\Delta V}{V} \)

**Ideal gas law**
\[ PV = n RT = N k T \]

**Ideal gas law**
\[ R = 8.314510 \text{ J/mol/K} = 0.0821 \text{ Latm/mol/K} \]

**Calorimetry:** \( \Delta Q = c m \Delta T \), \( \Delta Q = L \Delta m \)

**First law:** \( \Delta U = \Delta Q - \Delta W \), \( W = \int P dV \)

**Conduction:** \( H = \Delta Q - k A \Delta T, \quad \Delta T_{i} = \frac{-H}{k} \)

**Stefan’s law:**
\[ \sigma = \frac{e}{A}, \quad \sigma = 5.67 \times 10^{-8} \text{ W/m}^{2} \text{K}^{4} \]

**Kinetic theory of gas**
- **Ideal gas:** \( \Delta p_{\text{v}} = 2 m v_{x}, \quad F = \frac{m v_{x}^{2}}{2} \)

**Pressure:**
\[ P = \frac{N}{V}, \quad n N \frac{\mu}{2} = \frac{N}{V} \]

**Ideal gas law**
\[ P = \frac{n}{V}, \quad T x = \frac{k}{\mu}, \quad T = 273 + T_{c}, \quad PV = n k T \]

**Constant P:**
\[ \Delta Q = n C_{p} \Delta T \]

**γ:**
\[ C_{p} = \frac{C_{v}}{\gamma}, \quad C_{p} - C_{v} = \frac{R}{\gamma} \]

**C_{v} = \frac{R}{d}, \quad \text{for transl.+rot.+vib.}, \quad d = 3 + 2 + 2 \]

**Adiabatic expansion:**
\[ PV^{\gamma} = \text{constant} \]

**Mean free path:**
\[ \ell = \frac{v_{\text{rms}}}{\sqrt{2} \pi d^{2} n_{v}} = \frac{1}{\sqrt{2} \pi d^{2} n_{v}} \]
Airplane Momentum
11:03, calculus, numeric, > 1 min.

001
An airplane of mass 12000 kg flies level to the ground at an altitude of 10 km with a constant speed of 175 m/s relative to the Earth.

What is the magnitude of the airplane’s angular momentum relative to a ground observer directly below the airplane in kg·m²/s? Correct answer: 2.1 × 10¹⁰ kg·m²/s.

Explanation:
Since the observer is directly below the airplane, the perpendicular distance from the line of flight to Earth’s surface doesn’t change.

L = h m v

002
Does this value change as the airplane continues its motion along a straight line?

1. No. L = constant. correct
2. Yes. L increases as the airplane moves.
3. Yes. L decreases as the airplane moves.
4. Yes. L changes in a random pattern as the airplane moves.
5. Yes. L changes with certain period as the airplane moves.

Explanation:
L = constant since the perpendicular distance from the line of flight to Earth’s surface doesn’t change.

Mass on Solid Cylinder
11:03, calculus, numeric, > 1 min.

003
Given: g = 9.8 m/s².

A 4 kg mass is attached to a light cord, which is wound around a pulley. The pulley is a uniform solid cylinder of radius 8 cm and mass 1 kg.

What is the resultant net torque on the system about the center of the wheel? Correct answer: 3.136 kg m²/s².

Explanation:
The net torque on the system is the torque by the external force, which is the weight of the mass. So it is given by

\[\tau = r F \sin \phi = r m g \sin 90^\circ = r m g\]
\[= (0.08 \text{ m})(4 \text{ kg})(9.8 \text{ m/s}^2)\]
\[= 3.136 \text{ kg m}^2/\text{s}^2.\]

004
When the falling mass has a speed of 5 m/s, the pulley has an angular velocity of \(\frac{v}{r}\).

Determine the total angular momentum of the system about the center of the wheel. Correct answer: 1.8 kg m²/s.

Explanation:
The total angular momentum has two parts, one of the pulley and one of the mass. So it is

\[\|\vec{L}\| = \|\vec{r} \times m \vec{v} + I \vec{\omega}\|\]
\[= r m v + \frac{1}{2} M r^2 \left(\frac{v}{r}\right)\]
\[= r \left( m + \frac{M}{2} \right) v\]
\[= (8 \text{ cm}) \left[ (4 \text{ kg}) + \frac{(1 \text{ kg})}{2} \right] v\]
\[= (0.36 \text{ kg m}) v\]
\[= (0.36 \text{ kg m})(5 \text{ m/s})\]
\[= 1.8 \text{ kg m}^2/\text{s}.\]

005
Using the fact that \(\tau = dL/dt\) and your result from the previous part, calculate the acceleration of the falling mass. Correct answer: 8.71111 m/s².

Explanation:
Use the torque-angular momentum relation, we have

\[\tau = \frac{dL}{dt} = \frac{d}{dt}(0.36 \text{ kg m} v) = 0.36 \text{ kg m a},\]
Solving for acceleration

\[
a = \frac{\tau}{0.36 \text{ kg m}} = \frac{(3.136 \text{ kg m}^2/s^2)}{(0.36 \text{ kg m})} = 8.71111 \text{ m/s}^2.
\]

The equation of motion \( \tau = I\alpha \) gives:

\[
\frac{mgL}{2} = I\alpha = \frac{1}{3}mL^2\alpha,
\]

So

\[
\alpha = \frac{3g}{2L}.
\]

Meter Stick

11:04, calculus, multiple choice, \( > 1 \text{ min.} \)

A uniform meter-stick with length \( L \) pivots at point \( O \). The meter stick can rotate freely about \( O \). We release the stick from the horizontal position at \( t = 0 \).

Determine the angular acceleration immediately after the release of the stick. (Hint: Consider the equation of motion at \( t = 0 \).)

1. \( \frac{g}{4L} \)
2. \( \frac{g}{3L} \)
3. \( \frac{g}{2L} \)
4. \( 3g \)
5. \( \frac{5g}{6L} \)
6. \( \frac{g}{L} \)
7. \( \frac{5g}{4L} \)
8. \( \frac{3g}{2L} \) correct
9. \( \frac{7g}{4L} \)
10. \( \frac{2g}{L} \)

Explanation:

Assume the stick has a sufficient width, coins may be placed on the stick. Put coin 1 at \( P_1 \), where \( OP_1 = L/2 \), coin 2 at \( P_2 \), where \( OP_2 = 3L/4 \) and coin 3 at \( P_3 \), where \( OP_3 = L \). Now we release the stick from the horizontal position at the time \( t = 0 \).

Which coins are expected to stay on the stick immediately after the release of the stick? Assume when \( a_c \), the acceleration of the coin is greater than or equal to \( a_s \), the local acceleration of the meter stick, i.e., \( a_c \geq a_s \), the coin will stay on the stick. But when \( a_c < a_s \), the coin will be detached from (i.e., will not stay on) the stick. (Hint: Compare the linear acceleration of the segment of the stick beneath each coin.)

1. None will stay on the stick
2. Only coin 1 will stay on the stick correct
3. Only coin 2 will stay on the stick
4. Only coin 3 will stay on the stick
5. Only coin 1 and coin 2 will stay on the stick
6. Only coin 1 and coin 3 will stay on the stick
7. Only coin 2 and coin 3 will stay on the stick
8. All will stay on the stick

**Explanation:**
From part 1, we know the angular acceleration of the stick is
\[ \alpha = \frac{3g}{2L}. \]
The downward linear acceleration of the stick at \( P_1 \) is
\[ a_1 = \alpha \left( \frac{L}{2} \right) = \frac{3g}{4}. \]
At \( P_2 \), the downward linear acceleration is
\[ a_2 = \alpha \left( \frac{3L}{4} \right) = \frac{9g}{8}. \]
At \( P_3 \), the downward linear acceleration is
\[ a_3 = \alpha L = \frac{3g}{2}. \]

At the moment when the stick is released, the accelerations of all the coins are \( g \), so at coin 1, the corresponding segment of the stick falls slower than the coin, at coin 2 and coin 3, the corresponding segments of the stick fall off faster than the coins. This implies that only coin 1 will stay on the stick.

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**Rolling Disk**
11:04, calculus, multiple choice, > 1 min.

Consider the descending motion of a uniform disk with mass \( m \) and radius \( b \). It is rolling down vertically as indicated in the sketch.

\[ T = \frac{1}{2} ma. \] (3)

In the 3rd step we used \( a = \alpha b \). Substituting equation (3) into (1) leads to,
\[ mg - \frac{1}{2} ma = ma \]
\[ a = \frac{2}{3} g, \]
and
\[ T = \frac{1}{2} ma = \frac{1}{3} mg. \]
1. \( T = \frac{2}{5} mg \)

2. \( T = \frac{1}{2} mg \)

3. \( T = \frac{2}{3} mg \)

4. \( T = \frac{1}{3} mg \) correct

5. \( T = mg \)

6. \( T = \frac{3}{2} mg \)

7. \( T = \frac{3}{4} mg \)

**Explanation:**

Determine the descending speed \( v_p \) of its center, after the disk has dropped for a height \( h \).

1. \( v_p = \sqrt{\frac{4}{3} gh} \) correct

2. \( v_p = \sqrt{\frac{2}{3} gh} \)

3. \( v_p = \sqrt{\frac{1}{2} gh} \)

4. \( v_p = \sqrt{\frac{1}{3} gh} \)

5. \( v_p = \sqrt{\frac{3}{2} gh} \)

6. \( v_p = \sqrt{2 gh} \)

7. \( v_p = \sqrt{gh} \)

8. \( v_p = \sqrt{\frac{5}{3} gh} \)

**Explanation:**

Applying “\( v^2 - v_0^2 = 2as \)” to the motion of the center of mass, \( (v_0 = 0) \),

\[ v_P^2 = 2as = 2 \left( \frac{2}{3} g \right) h \]

\[ \Rightarrow \ v_P = \sqrt{\frac{4}{3} gh}. \]

**Alternative method:**

The application of work-energy theorem gives:

\[ mgh = \frac{1}{2} mv_P^2 + \frac{1}{2} I \omega_P^2. \]

But

\[ \frac{1}{2} I \omega_P^2 = \frac{1}{2} \left( \frac{1}{2} mb^2 \right) \omega_P^2 = \frac{1}{4} mv_P^2. \]

So

\[ mgh = \left( \frac{1}{2} + \frac{1}{4} \right) mv_P^2. \]

\[ \Rightarrow \ v_P = \sqrt{\frac{4}{3} gh}. \]

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**Clay Rotates a Rod**

11:05, calculus, multiple choice, > 1 min.

A uniform rod, supported and pivoted at its midpoint, but initially at rest, has a mass \( 2m \) and a length \( l \). A piece of clay with mass \( m \) and velocity \( v \) hits the very top of the rod, gets stuck and causes the rod-clay system to spin about the pivot point \( O \) in a horizontal plane. Viewed from above the scheme is
With respect to the pivot point O, what is the magnitude of the initial angular momentum $L_i$ of the piece of clay and the final moment of inertia $I_f$ of the clay-rod system?

1. $L_i = m v l, \quad I_f = \frac{5}{12} ml^2$
2. $L_i = m v \frac{l}{2}, \quad I_f = \frac{8}{12} ml^2$
3. $L_i = m v l, \quad I_f = \frac{3}{12} ml^2$
4. $L_i = m v l, \quad I_f = \frac{8}{12} ml^2$
5. $L_i = m v l, \quad I_f = \frac{4}{12} ml^2$
6. $L_i = m v \frac{l}{2}, \quad I_f = \frac{5}{12} ml^2$ \textbf{correct}
7. $L_i = m v \frac{l}{2}, \quad I_f = \frac{4}{12} ml^2$
8. $L_i = m v \frac{l}{2}, \quad I_f = \frac{7}{12} ml^2$

**Explanation:**

**Basic Concepts:** Conservation of angular momentum $\vec{L}$

\[
\vec{L} = \vec{r} \times \vec{p} = m \vec{r} \times \vec{v}
\]

\[
\sum \vec{\tau}_{ext} = \frac{d \vec{L}}{dt}
\]

$L_z = I \omega$.

Therefore, if the net external torque acting on a system is zero, the total angular momentum of that system is constant.

Since the total external torque acting on the system clay-rod is zero, the total angular momentum is a constant of motion. The total initial angular momentum $L_i$ is simply the angular momentum of the clay, since the rod is at rest initially

\[
L_i = \|\vec{r} \times \vec{p}\| = m r v = \frac{m v l}{2}.
\]

The final moment of inertia $I_f$ of the clay-rod system is the moment of inertia of the rod plus the moment of inertia of the clay

\[
I_f = I_{rod} + I_{clay} = \frac{1}{12} 2 ml^2 + m \left( \frac{l}{2} \right)^2 = \frac{5}{12} ml^2.
\]

The final angular speed $\omega_f$ of the rod-clay system is

1. $\omega_f = \frac{6}{5} v$.
2. $\omega_f = \frac{12}{5} v$ \textbf{correct}
3. $\omega_f = \frac{6}{5} \frac{v}{l}$.
4. $\omega_f = \frac{4}{6} v$.
5. $\omega_f = \frac{12}{7} v$ \textbf{correct}
6. $\omega_f = \frac{5}{6} v$.
7. $\omega_f = \frac{6}{2} v$.
8. $\omega_f = \frac{5}{12} v$.
9. $\omega_f = \frac{12}{7} v$.
10. $\omega_f = \frac{3}{5} v$.

**Explanation:**

The total final angular momentum is the same as the total initial angular momentum. According to $L = I \omega$, we have

\[
L_f = L_i
\]
\[ m v l = I_f \omega_f \]
\[ \omega_f = \frac{6 \nu}{5 l} \]

**Bohr Model of Hydrogen 02**

11:07, calculus, numeric, > 1 min.

**013**

In the Bohr’s model of the hydrogen atom, the electron moves in a circular orbit of radius \(5.29 \times 10^{-11}\) m around the proton. Assume that the orbital angular momentum of the electron is equal to \(h\).

Calculate the orbital speed of the electron.

Correct answer: \(1.37503 \times 10^7\) m/s.

**Explanation:**

The angular momentum of the electron in the ground state of the hydrogen atom (this the case here) in the Bohr’s model is \(h\), therefore:

\[ L = m v r = h \]

and solving for \(v\),

\[ v = \frac{h}{m r} \]
\[ = \frac{6.62608 \times 10^{-34} \text{ J s}}{9.10939 \times 10^{-31} \text{ kg}(5.29 \times 10^{-11} \text{ m})} \]
\[ = 1.37503 \times 10^7 \text{ m/s} \]

**014**

Calculate the angular frequency of the electron’s motion.

Correct answer: \(2.5993 \times 10^{17}\) s\(^{-1}\).

**Explanation:**

The angular frequency is given by

\[ \omega = \frac{v}{r} \]
\[ = \frac{(1.37503 \times 10^7 \text{ m/s})}{(5.29 \times 10^{-11} \text{ m})} \]
\[ = 2.5993 \times 10^{17} \text{ s}^{-1} \]

---

**015**

Two objects, of masses 6 and 8 kilograms, are hung from the ends of a stick that is 70 centimeters long and has marks every 10 centimeters, as shown.

If the mass of the stick is negligible, at which of the points indicated should a cord be attached if the stick is to remain horizontal when suspended from the cord?

1. A
2. B
3. C
4. D correct
5. E

**Explanation:**

For static equilibrium, \(\tau_{net} = 0\).

Denote \(x\) the distance from the left end point of the stick to the point where the cord is attached.

\[ 0 = \tau = (6) g x - (8) g (70 - x) \]
\[ \implies 6 x - 8 (70 - x) = 0 \]
\[ \implies 14 x = 560 \]
\[ \implies x = 40 \text{ cm.} \]

Therefore the point should be point \(D\).

---

**Equilibrium of Hinged Lever 01**

12:01, calculus, multiple choice, > 1 min.

**016**

A uniform rod pivoted at one end, point \(O\), is free to swing in a vertical plane in a gravitational field. However, it is held in equilibrium by a force \(F\) at its other end.
Force vectors are drawn to scale.
What is the condition for translational equilibrium along the horizontal $x$ direction?

1. $-R_x + F_x = 0$ **correct**
2. $F_x = 0$
3. $R_x - F_x \cos \theta = 0$
4. $R_x - F_x \sin \theta = 0$
5. $F_x \cos \theta - R_x \sin \theta = 0$

**Explanation:**
Using the distances, angles and forces depicted in the figure, the condition $\sum F_x = 0$ for translational equilibrium in the $x$ direction is

$$-R_x + F_x = 0.$$ 

What is the condition for translational equilibrium along the vertical $y$ direction?

1. $R_y + F_y - W = 0$ **correct**
2. $R_y + F_y = 0$
3. $R_y - F_y + W = 0$
4. $W - R_y + F_y = 0$
5. $R_y \sin \theta + F_y \sin \theta - W \cos \theta = 0$

**Explanation:**
For the equilibrium in the $y$ direction, $\sum F_y = 0$ turns out to be

$$R_y + F_y - W = 0.$$

Taking the origin of the torque equation at point $O$, what is the condition for rotational equilibrium?

1. $F_y \ell \cos \theta - W \frac{\ell}{2} \cos \theta - F_x \ell \sin \theta = 0$ **correct**
2. $2 F_y \ell \sin \theta - W \ell \cos \theta - 2 F_x \ell \sin \theta = 0$
3. $F_y \ell \cos \theta - W \ell \sin \theta + F_x \ell \sin \theta = 0$
4. $F_y \ell \sin \theta - W \frac{\ell}{2} \sin \theta - F_x \ell \sin \theta = 0$
5. $W \frac{\ell}{2} - F_x \ell \cos \theta - F_y \ell \cos \theta = 0$

**Explanation:**
For rotational equilibrium about point $O$, the net torque on the system about that point must vanish. The angle $\theta$ appears as follows and we see that the forces $R_x$ and $R_y$ exert no torque on the point $O$. From the figure we have, counterclockwise

$$\ell F_y \cos \theta - \frac{\ell}{2} W \cos \theta - \ell F_x \sin \theta = 0.$$ 

**Tilting a Block**
12:02, calculus, numeric, > 1 min.

**019**

Given: $g = 9.8 \text{ m/s}^2$.
Consider the rectangular block of mass $m = 40 \text{ kg}$ height $h = 1 \text{ m}$, length $l = 0.6 \text{ m}$. A
force $F$ is applied horizontally at the upper edge.

What is the minimum force required to start to tip the block?
Correct answer: 117.6 N.

**Explanation:**

**Basic Concepts:** In equilibrium
\[
\sum \vec{F} = 0
\]
\[
\sum \vec{\tau} = 0
\]

**Part 1:** Let the right lower corner of the block be denoted as $A$. Then
\[
\sum \tau_A = h F - \frac{l}{2} m g = 0
\]
Therefore
\[
F = \frac{m g l}{2 h}
\]
\[
= (40 \text{ kg})(9.8 \text{ m/s}^2) \frac{0.6 \text{ m}}{2(1 \text{ m})}
\]
\[
= 117.6 \text{ N}.
\]

Solving for $\mu$,
\[
\mu = \frac{F}{N_A}
\]
\[
= \frac{F}{m g}
\]
\[
= \frac{117.6 \text{ N}}{(40 \text{ kg})(9.8 \text{ m/s}^2)}
\]
\[
= 0.3
\]

---

**Bricks on the Brink**

A uniform brick of length 20 cm is placed over the edge of a horizontal surface with the maximum overhang $x$ possible without falling.

Find $x$ for a single block.
Correct answer: 10 cm.

**Explanation:**

**Basic Concepts:** The definition of the center of mass (n bricks):
\[
x_{cm} \equiv \frac{\sum_{i=1}^{n} x_i m_i}{\sum_{i=1}^{n} m_i} = \frac{1}{n} \sum_{i=1}^{n} x_i
\]
where $x_i$ is the center of mass position of the $i^{th}$ brick and $m_i$ is the mass of the $i^{th}$ brick.

**Solution:** The center of mass of a single brick is in its middle or $\frac{1}{2}$ of a brick’s length from its maximum overhang. Since $\frac{x_1}{L} = \frac{1}{2}$, as measured from the maximum overhang,
\[
\frac{x_{cm}}{L} \bigg|_{n=1} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}
\]

---

Two identical uniform bricks of length 20 cm
are stacked over the edge of a horizontal surface with the maximum overhang \( x \) possible without falling.

Find \( x \) for two blocks.
Correct answer: 15 cm.

**Explanation:**

**Basic Concepts:** Since \( m_i = m \) (all bricks have the same mass), \( \sum_{i=1}^{n} m = n m \). Note:

The bricks will just balance when the center of mass is over the fulcrum; \( i.e. \), the edge of the horizontal surface. Note: In the solution below, measurement will be made from the left edge of the top brick with the maximum overhang. To calculate the center of mass

\[
\frac{x_{cm}}{L} = \frac{1}{n} L \sum_{i=1}^{n} x_i,
\]

when an additional brick is positioned at the bottom of the stack (\( n \) bricks), the additional brick’s edge is placed at the edge of the horizontal surface of the previous stack (\( n-1 \) bricks). The center of mass of the additional brick is \( \frac{1}{2} \) of a brick’s length plus the maximum overhang of the previous stack (\( n-1 \) bricks).

**Solution:** The top brick can extend \( \frac{1}{2} \) of a brick’s length from the maximum overhang. When the top brick extends \( \frac{1}{2} \) its length past the second brick, the center of mass of the top two bricks is in their middle or \( \frac{3}{4} \) of a single brick’s length from the maximum overhang.

Since

\[
\frac{x_{cm}}{L} = \left. \frac{x_{cm}}{L} \right|_{n=2} \right| \frac{1}{2} + \frac{1}{2} = 1
\]

as measured from the maximum overhang

\[
\left. \frac{x_{cm}}{L} \right|_{n=2} = \frac{1}{2} + \frac{1}{2} = \frac{3}{4}.
\]

\[023\]

Three identical uniform bricks of length 20 cm are stacked over the edge of a horizontal surface with the maximum overhang \( x \) possible without falling.

Find \( x \) for three blocks.
Correct answer: 18.3333 cm.

**Explanation:**

The top two bricks can extend \( \frac{3}{4} \) of a brick’s length from the maximum overhang. When the top two bricks extended \( \frac{3}{4} \) of their length past the third brick, the center of mass of the top three bricks is in their middle or \( \frac{11}{12} \) of a brick’s length from the maximum overhang.

Since

\[
\frac{x_{cm}}{L} \right|_{n=3} = \frac{2}{3} + \frac{3}{4} = \frac{11}{12}.
\]

\[024\]

\( n \) identical uniform bricks of length \( L \) are stacked over the edge of a horizontal surface with the maximum overhang \( x \) possible without falling.

Find \( x \) for \( n \) blocks.

1. \( \frac{L}{2}\sum_{i=1}^{n} \frac{1}{2i - 1} \)
2. \( L\sum_{i=1}^{n} \frac{1}{i + 1} \)
3. \( L\sum_{i=1}^{n} \frac{1}{2^i} \)
4. \( \frac{3L}{2}\sum_{i=1}^{n} \frac{1}{i + 2} \)
5. \( \frac{L}{2} \sum_{i=1}^{n} \frac{1}{i} \) correct

6. \( \frac{L}{2} \sum_{i=1}^{n} \frac{1}{i! + 1} \)

7. \( \frac{L}{2} \sum_{i=1}^{n} \left( \frac{1}{i+1} + \frac{1}{4} \right) \)

8. \( \frac{L}{2} \sum_{i=1}^{n} \frac{1}{i!} \)

9. \( \frac{L}{2} \sum_{i=1}^{n} \frac{i}{2!} \)

Explanation:

We can rewrite the first three solutions:

\[
\frac{x_{cm}}{L} \bigg|_{n=1} = \frac{1}{2} = \frac{1}{2} (1) \\
\frac{x_{cm}}{L} \bigg|_{n=2} = \frac{3}{4} = \frac{1}{2} \left( \frac{3}{2} \right) = \frac{1}{2} \left( 1 + \frac{1}{2} \right) \\
\frac{x_{cm}}{L} \bigg|_{n=3} = \frac{11}{12} = \frac{1}{2} \left( \frac{11}{6} \right) \\
= \frac{1}{2} \left( \frac{6}{6} + \frac{3}{6} + \frac{2}{6} \right) \\
= \frac{1}{2} \left( 1 + \frac{1}{2} + \frac{1}{3} \right). 
\]

For four bricks, the top three bricks can extend \( \frac{11}{12} \) of a brick’s length from the maximum overhang. When the top three bricks extend \( \frac{11}{12} \) of their length past the fourth brick, the center of mass of the top three bricks is in their middle or \( \frac{25}{24} \) of a brick’s length from the maximum overhang. Since

\[
\frac{x_{4}}{L} = \frac{x_{cm}}{L} \bigg|_{n=3} + \frac{1}{2} = \frac{11}{12} + \frac{1}{2} = \frac{17}{12}, 
\]

as measured from the maximum overhang,

\[
\frac{x_{cm}}{L} \bigg|_{n=4} = \frac{\frac{1}{2} + 1 + \frac{5}{4} + \frac{17}{12}}{4} \\
= \frac{25}{24} = \frac{1}{2} \left( \frac{25}{12} \right) \\
= \frac{1}{2} \left( \frac{12}{12} + \frac{6}{12} + \frac{4}{12} + \frac{3}{12} \right) \\
= \frac{1}{2} \left( 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \right). 
\]

For five bricks,

\[
\frac{x_{cm}}{L} \bigg|_{n=4} + \frac{1}{2} = \frac{25}{24} + \frac{1}{2} = \frac{37}{24}, 
\]

\[
\frac{x_{cm}}{L} \bigg|_{n=5} = \frac{1}{2} + 1 + \frac{5}{4} + \frac{17}{12} + \frac{37}{24} \\
= \frac{137}{120} = \frac{1}{2} \left( \frac{137}{60} \right) \\
= \frac{1}{2} \left( \frac{60 + 30 + 20 + 15 + 12}{60} \right) \\
= \frac{1}{2} \left( 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} \right). 
\]

For six bricks,

\[
\frac{x_{cm}}{L} \bigg|_{n=4} + \frac{1}{2} = \frac{137}{120} + \frac{1}{2} = \frac{197}{120}, 
\]

\[
\frac{x_{cm}}{L} \bigg|_{n=6} = \frac{1}{2} + 1 + \frac{5}{4} + \frac{17}{12} + \frac{37}{24} + \frac{197}{120} \\
= \frac{441}{180} = \frac{1}{2} \left( \frac{441}{60} \right) \\
= \frac{1}{2} \left( \frac{180 + 90 + 60 + 45}{180} + \frac{36}{180} \right) \\
= \frac{1}{2} \left( 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} \right). 
\]

And so forth...

Thus for \( n \) bricks, we have

\[
\frac{x_{cm}}{L} \bigg|_{n \text{ bricks}} = \frac{1}{2} \left( 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} \right) \\
= \frac{1}{2} \sum_{i=1}^{n} \frac{1}{i}. 
\]

Alternate Perspective: Rewriting the sequence for the \( x_{cm} \)‘s we have

\[
\frac{x_{cm}}{L} = \frac{1}{2} \left( 1, 1 + \frac{1}{2}, \right. \\
1 + \frac{1}{2}, \\
1 + \frac{1}{2}, \\
\cdots \\
1 + \frac{1}{2}, \\
\left. 1 + \frac{1}{2} \right). 
\]
\[
1 + \frac{1}{2} + \frac{1}{3},
\]
\[
1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4},
\]
\[
1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5},
\]
\[
1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6},
\]
\[
= \frac{1}{2} \left(1, \frac{3}{2}, \frac{11}{6}, \frac{25}{12}, \frac{137}{60}, \ldots\right)
\]
\[
= \frac{1}{2} \left(1, \frac{3}{2}, \frac{11}{6}, \frac{25}{12}, \frac{137}{60}, \ldots\right)
\]
\[
= \frac{1}{2} \left(\sum_{i=1}^{n} \frac{1}{i}\right).
\]
So \(n\) bricks (i.e. using \(\frac{1}{2}\) times the sum of \(i\) elements in the series, \(\frac{1}{i}\)), we have
\[
\frac{x_{\text{cm}}}{L} \bigg|_{n \text{ bricks}} = \frac{1}{2} \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \cdots\right)
\]
\[
= \frac{1}{2} \sum_{i=1}^{n} \frac{1}{i}.
\]

Note: The maximum overhang occurs as given, that is, when the top block is extended as far out as possible on the second block, and the top two blocks are extended as far out as possible on the third block, and so forth. Other mechanical arrangements do not give a maximum overhang. Furthermore, this series does not converge, so any overhang is possible. A careful classroom demonstration using only four bricks will show the top brick completely past the edge of the horizontal surface (\(x = L\)). How many bricks are required to have the overhang \(x = 2L\)?

Dishonest Shopkeeper

Two pans of a balance are 50 cm apart. The fulcrum of the balance has been shifted 1 cm away from the center by a dishonest shopkeeper.

By what percentage is the true weight of the goods being marked up by the shopkeeper?

(\(\text{Assume the balance has negligible mass.}\))

Correct answer: 8.33334 percent.

**Explanation:**

- \(W\) → standard weight
- \(W'\) → weight of goods sold
- \(W(L/2 - l) = W'(L/2 + l)\)

Therefore

\[
W = W' \left(\frac{L/2 + l}{L/2 - l}\right)
\]

and

\[
\left(\frac{W - W'}{W'}\right) \times 100
\]

\[
= \left(\frac{L/2 + l}{L/2 - l} - 1\right) \times 100
\]

\[
= \left(\frac{(50 \text{ cm})/2 + (1 \text{ cm})}{(50 \text{ cm})/2 - (1 \text{ cm})} - 1\right) \times 100
\]

\[
= 8.33334 \text{ percent}
\]

---

**Elongation of a Rod**

In the figure the radius of the rod is 0.2 cm, the length of aluminum part is 1.3 m, the copper part is 2.6 m. For aluminum, Young’s modulus is \(7 \times 10^{10} \text{ Pa}\), for copper, \(1.1 \times 10^{11} \text{ Pa}\).

Determine the elongation of the rod if it is under a tension of 5800 N

Correct answer: 1.9481 cm.

**Explanation:**

The cross-sectional area of the wire \(A = \pi (0.2 \text{ cm})^2 = 1.25664 \times 10^{-5} \text{ m}^2\), and the tension = 5800 N N throughout both pieces. Let us compute the elongation of each part separately: For aluminum:

\[
\Delta L_{al} = \frac{L_0 F}{Y A}
\]

\[
= \frac{(1.3 \text{ m})(5800 \text{ N})}{(7 \times 10^{10} \text{ Pa})(1.25664 \times 10^{-5} \text{ m}^2)}
\]

\[
= 0.00857163 \text{ m}.
\]
Similarly, for the copper part:

\[
\Delta L_{\text{cu}} = \frac{L_0 F}{YA}
\]

\[
= \frac{(2.6 \text{ m})(5800 \text{ N})}{(1.1 \times 10^{11} \text{ Pa})(1.25664 \times 10^{-5} \text{ m}^2)}
\]

\[= 0.0109093 \text{ m}.\]

So, the total elongation \[= 0.019481 \text{ m} = 1.9481 \text{ cm} \].

---

**Serway CP 09 02**
1:24, trigonometry, numeric, > 1 min.

If the shear stress in steel exceeds about \[4 \times 10^8 \text{ N/m}^2\], the steel ruptures. Find the shearing force necessary to shear a steel bolt 1 cm in diameter.

Correct answer: 31415.9 N.

**Explanation:**

Given: \[r = 0.5 \text{ cm} = 0.005 \text{ m}\] and \[\text{Stress} = 4 \times 10^8 \text{ N/m}^2\].

\[F = A \times \text{Stress} \]

\[= 2 \pi r t \times \text{Stress} \]

\[= 2 \pi (0.005 \text{ m})(0.005 \text{ m}) \cdot (4 \times 10^8 \text{ N/m}^2) \]

\[= 62831.9 \text{ N}.\]

---

**Force on a Table**
15:01, trigonometry, numeric, > 1 min.

Given: \[g = 9.8 \text{ m/s}^2\].

A 0.75 kg physics book with dimensions of 24 cm by 20 cm is on a table. What force does the book apply to the table?

Correct answer: 7.35 N.

**Explanation:**

\[F = mg\]

---

Find the shearing force necessary to punch a hole 1 cm in diameter in a steel plate 0.5 cm thick.

Correct answer: 62831.9 N.

**Explanation:**

Given: \[t = 0.5 \text{ cm} = 0.005 \text{ m}\].

---

Holt SF 09Rev 19
15:01, highSchool, numeric, > 1 min.

Given: \[g = 9.81 \text{ m/s}^2\].

A submarine is at an ocean depth of 250 m. Assume that the density of sea water is \[1.025 \times 10^3 \text{ kg/m}^3\] and the atmospheric pressure is \[1.01 \times 10^5 \text{ Pa}\].

a) Calculate the absolute pressure at this depth.

Correct answer: \[2.61481 \times 10^6 \text{ Pa}\].
Explanation:

Basic Concept:

\[ P = P_0 + \rho gh \]

Given:

\[ h = 250 \text{ m} \]
\[ \rho_{\text{sea water}} = 1.025 \times 10^3 \text{ kg/m}^3 \]
\[ g = 9.81 \text{ m/s}^2 \]
\[ P_0 = 1.01 \times 10^5 \text{ Pa} \]

Solution:

\[ P = 101000 \text{ Pa} \]
\[ + (1025 \text{ kg/m}^3) \]
\[ \cdot (9.81 \text{ m/s}^2)(250 \text{ m}) \]
\[ = 2.61481 \times 10^6 \text{ Pa} \]

b) Calculate the magnitude of the force exerted by the water at this depth on a circular submarine window with a diameter of 30.0 cm.
Correct answer: 184830 N.

Explanation:

Basic Concepts:

\[ P = \frac{F}{A} \]

\[ A = \pi r^2 = \pi \left( \frac{D}{2} \right)^2 \]

Given:

\[ D = 30.0 \text{ cm} \]

Solution:

\[ F = PA \]
\[ = P \left( \frac{\pi D^2}{4} \right) \]
\[ = (2.61481 \times 10^6 \text{ Pa}) \frac{\pi (30 \text{ cm})^2}{4} \]
\[ \cdot \left( \frac{1 \text{ m}}{100 \text{ cm}} \right)^2 \]
\[ = 184830 \text{ N} \]