1. QCD Evolution of PDFs
2. Measurement of $\sin^2\theta_W$
3. Formalism of $\sin^2\theta_W$ in $\nu$-N DIS
4. Improvements in $\sin^2\theta_W$
5. Interpretation and Link to Higgs
Factorization

\[ \sigma = f \sigma_p \]

Non-perturbative, infra-red part

Partonic hard scatter

\( s = f_s s_p \)
DGLAP QCD Evolution Equations

- The evolution equations by Dokshitzer-Gribov-Lipatov-Altarelli-Parisi provide mechanism to evolve PDF’s to any kinematic regime or momentum scale.

\[
\frac{dq^{NS}(x,M^2)}{d\ln M^2} = \sum_i \frac{d_q^i - q}{-q} = u_v + d_v = \frac{\alpha_s(\mu^2)}{2\pi} \int_x^1 \frac{dy}{y} q^{NS}(y,M^2) P_{qq}(\frac{x}{y})
\]

\[
\frac{dq^S(x,M^2)}{d\ln M^2} = \sum_i \frac{d_q^i + q}{q} = \frac{\alpha_s(\mu^2)}{2\pi} \int_x^1 \frac{dy}{y} \left[ q^{NS}(y,M^2) P_{qq}(\frac{x}{y}) + G(y,M^2) P_{qq}(\frac{x}{y}) \right]
\]

\[
\frac{dG(x,M^2)}{d\ln M^2} = \frac{\alpha_s(\mu^2)}{2\pi} \int_x^1 \frac{dy}{y} \left[ q^S(y,M^2) P_{Gq}(\frac{x}{y}) + G(y,M^2) P_{GG}(\frac{x}{y}) \right]
\]

\(P_q(x/y):\) Splitting function that is the probability of parton i with momentum y get resolved as parton j with momentum x<y
Feynman Diagrams for Parton Splitting

LO: $O(\alpha_s)$

NLO: $O(\alpha_s^2)$
Electroweak Theory

- Standard Model unifies Weak and EM to SU(2) x U(1) gauge theory
  - Weak neutral current interaction
  - Measured physical parameters related to mixing parameters for the couplings

\[
g' = g \tan \theta_w, \quad e = g \sin \theta_w, \quad G_F = \frac{g^2 \sqrt{2}}{8M_w^2}, \quad \frac{M_w}{M_z} = \cos \theta_w
\]

- Neutrinos in this picture are unique because they only interact through left-handed weak interactions \( \rightarrow \) Probe weak sector only
  - Less complication in some measurements, such as proton structure
**sin^2\theta_W** and \(\nu-N\) scattering

- In the electroweak sector of the Standard Model, it is not known *a priori* what the mixture of electrically neutral electromagnetic and weak mediator is. This fractional mixture is given by the mixing angle.

- Within the on-shell renormalization scheme, \(\sin^2\theta_W\) is:

\[
\sin^2 \theta_W^{\text{On-Shell}} = 1 - \frac{M_W^2}{\rho_0 M_Z^2}
\]

- Provides independent measurement of \(M_W\) & information to pin down \(M_{\text{Higgs}}\) via higher order loop corrections, in comparable uncertainty to direct measurements.

- Measures light quark couplings \(\Rightarrow\) Sensitive to other types (anomalous) of couplings.

- In other words, sensitive to physics beyond SM \(\Rightarrow\) New vector bosons, compositeness, \(\nu\)-oscillations, etc.
Higher Order Corrections

- LO GSW requires three parameters: $\alpha$, $G_F$ and $M_Z$
- Higher order corrections bring in dependence two additional parameters: $M_{\text{Top}}$ and $M_{\text{Higgs}}$

$$\sigma \propto O(\alpha_W^4)$$
How is $\sin^2\theta_W$ measured?

- Cross section ratios between NC and CC proportional to $\sin^2\theta_W$
- Llewellyn Smith Formula:

$$R^{v(\nu)} = \frac{S^{v(\nu)}_{NC}}{S^{v(\nu)}_{CC}} = \sin^2 \theta_W \left( \frac{1}{2} - \sin^2 \theta_W \right) + \frac{5}{9} \sin^4 \theta_W \left( 1 + \frac{S^{v(\nu)}_{CC}}{S^{v(\nu)}_{CC}} \right)$$
Original Experiments

E770: Quad Triplet Beam and Lab E Detector

- Conventional neutrino beam from $\pi/k$ decays
- Focus all signs of $\pi/k$ for neutrinos and antineutrinos
- Only $\nu_\mu$ in the beam (NC events are mixed)

- Very small cross section $\Rightarrow$ Heavy neutrino target
- $\nu_e$ are the killers (CC events look the same as NC events)
How Can Events be Separated?

Charged Current Events

Neutral Current Events

Nothing is coming in!!!

Nothing is going out!!!
Experimental Variable

Define an Experimental Length variable

\( \Rightarrow \) Distinguishes CC from NC experimentally in statistical manner

\[
\begin{align*}
\text{Candidates CC} & \quad \text{Candidates NC} \\
\text{Cut} & \quad \text{Cut} \\
\text{Long} & \quad \text{Short} \\
\text{Exp} & \quad \text{N} \\
\text{LL} & \quad \text{LL} \\
\text{N} & \quad \text{NR} \\
\Rightarrow & \quad \leq \Rightarrow \\
\end{align*}
\]

Compare experimentally measured ratio

\[
R_{\text{Exp}} = \frac{N_{\text{Short}}}{N_{\text{Long}}} = \frac{L < L_{\text{Cut}}}{L > L_{\text{Cut}}} = \frac{N_{\text{NC Candidates}}}{N_{\text{CC Candidates}}}
\]

to theoretical prediction of \( R^{\nu} \)
Past Experimental Results

\[
\sin^2 \theta^\text{On-Shell}_W = 1 - \frac{M_W^2}{M_Z^2} = 0.2277 \pm 0.0036
\]

\[\Rightarrow M_{W \text{ On-Shell}} = 80.14 \pm 0.19 \text{GeV/c}^2\]

The yellow band represents a correlated uncertainty!!
Improvements on Measurements

• Asses the uncertainties from previous measurements
• Determine what the sources of largest theoretical and experimental uncertainties are
• Provide new methods to reduce large uncertainties
\[ \sin^2\theta_W \] Theoretical Uncertainty

- Significant correlated error from CC production of charm quark \((m_c)\) modeled by slow rescaling mechanism

\[
\frac{R^-}{R^+} = \frac{s_{NC}^\nu - s_{NC}^\bar{\nu}}{s_{CC}^\nu - s_{CC}^\bar{\nu}} = \left(1 - \sin^2\theta_W \right) = \frac{R^\nu - R^\bar{\nu}}{1 - r}
\]

- Suggestion by Paschos-Wolfenstein by separating \(\nu\) and \(\bar{\nu}\) beams:

  \(\rightarrow\) Reduce charm CC production error by subtracting sea quark contributions
  \(\rightarrow\) Only valence u, d, and s contributes while sea quark contributions cancel out
  \(\rightarrow\) Massive quark production through Cabbio suppressed d, quarks only
Homework Assignments

• Draw a few additional Feynman diagrams for higher order GSW corrections to $\nu$-N scattering at the same order as those on pg 7 in this lecture
  – Due: One week from today, Wed., Feb. 5