PHYS 5326 – Lecture #20

Monday, Apr. 7, 2003
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• Super Symmetry Breaking
• MSSM Higgs and Their Masses
• Upper limit on $M_h$
Minimal Supersymmetric Model (MSSM)

Uses the same SU(3) x SU_L(2) x U_Y(1) gauge symmetry as the Standard Model and yields the following list of particles:

<table>
<thead>
<tr>
<th>Superfield</th>
<th>SU(3)</th>
<th>SU(2)_L</th>
<th>U(1)_Y</th>
<th>Particle Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>3</td>
<td>2</td>
<td>1/3</td>
<td>(u_L, d_L), (\tilde{u}_L, \tilde{d}_L)</td>
</tr>
<tr>
<td>\tilde{U}</td>
<td>3</td>
<td>1</td>
<td>-4/3</td>
<td>\tilde{u}_R, \tilde{d}_R</td>
</tr>
<tr>
<td>\tilde{D}</td>
<td>3</td>
<td>1</td>
<td>2/3</td>
<td>\tilde{d}_R, \tilde{\nu}_R</td>
</tr>
<tr>
<td>\tilde{L}</td>
<td>1</td>
<td>2</td>
<td>-1</td>
<td>(\nu_L, e_L), (\tilde{\nu}_L, \tilde{e}_L)</td>
</tr>
<tr>
<td>\tilde{E}</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>\tilde{\tau}_R, \tilde{\nu}_R</td>
</tr>
<tr>
<td>\Phi_1</td>
<td>1</td>
<td>2</td>
<td>-1</td>
<td>(\Phi_1, \tilde{h}_1)</td>
</tr>
<tr>
<td>\Phi_2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>(\Phi_2, \tilde{h}_2)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Superfield</th>
<th>SU(3)</th>
<th>SU(2)_L</th>
<th>U(1)_Y</th>
<th>Particle Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>G^a</td>
<td>8</td>
<td>1</td>
<td>0</td>
<td>g, \tilde{g}</td>
</tr>
<tr>
<td>W^3</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>W^3, \tilde{\omega}_3</td>
</tr>
<tr>
<td>\tilde{B}</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>B, \tilde{b}</td>
</tr>
</tbody>
</table>

Chiral Superfield

Vector Superfield

Majorana fermion partners
Higgs Sector in MSSM

In SM $L$ for EW interactions, fermion masses are generated by the Yukawa terms in $L$

$$L = -\lambda_d \overline{Q_L} \Phi d_R + \lambda_u \overline{Q_L} \Phi^c u_R + h.c.$$  

In MSSM, the term proportional to $\Phi^c = -i \tau_2 \Phi^*$ is not allowed, causing an introduction of another scalar doublet to give $\tau_3 = 1$ for SU(2)$_L$ fermion doublet mass.

Thus, MSSM has two higgs doublets, $\Phi_1$ and $\Phi_2$. 
SupersymmetricScalarPotential

Through the requirement of supersymmetric gauge invariance and demand for perturbative algebra to be valid, the scalar potential in MSSM is

\[ V = |\mu|^2 (|\Phi_1|^2 + |\Phi_2|^2) + \frac{g^2}{8} \left[ (|\Phi_1|^2 - |\Phi_2|^2)^2 + \frac{g^2}{2} |\Phi_1^* \cdot \Phi_2|^2 \right] \]

This potential has its minimum at \(<\Phi_1^0> = <\Phi_2^0> = 0\), giving \(<V> = 0\), resulting in no EW symmetry breaking.

It is difficult to break supersymmetry but we do know it must be broken.
Soft Supersymmetry Breaking

The simplest way to break SUSY is to add all possible soft (scale ~ $M_W$) supersymmetry breaking masses for each doublet, along with arbitrary mixing terms, keeping quadratic divergences under control.

The scalar potential involving Higgs becomes

$$V_H = \left(|\mu|^2 + m_1^2\right)|\Phi_1|^2 + \left(|\mu|^2 + m_2^2\right)|\Phi_2|^2 - \mu B \varepsilon_{ij} (\Phi_i^i + \Phi_j^j + h.c)$$

$$+ \frac{g^2 + g'^2}{8} \left(|\Phi_1|^2 - |\Phi_2|^2\right)^2 + \frac{g^2}{2} |\Phi_1^* \cdot \Phi_2|^2$$

The quartic terms are fixed in terms of gauge couplings therefore are not free parameters.
Higgs Potential of the SUSY

The Higgs potential in SUSY can be interpreted as to be dependent on three independent combinations of parameters

\[ |\mu|^2 + m_1^2; \quad |\mu|^2 + m_2^2; \quad \mu B \]

Where B is a new mass parameter.

If \( \mu B \) is 0, all terms in the potential are positive, making the minimum, \( <V> = 0 \), back to \( <\Phi_1^0> = <\Phi_2^0> = 0 \).

Thus, all three parameters above should not be zero to break EW symmetry.
SUSY Breaking

Symmetry is broken when the neutral components of the Higgs doublets get vacuum expectation values:

$$\langle \Phi_1 \rangle \equiv v_1; \quad \langle \Phi_2 \rangle \equiv v_2$$

The values of $v_1$ and $v_2$ can be made positive, by redefining Higgs fields.

When the EW symmetry is broken, the W gauge boson gets a mass which is fixed by $v_1$ and $v_2$.

$$M_W^2 = \frac{g}{2} \left( v_1^2 + v_2^2 \right)$$
SUSY Higgs Mechanism

Before the EW symmetry was broken, the two complex SU(2)$_L$ Higgs doublets had 8 DoF of which three have been are observed to give masses to W and Z gauge bosons, leaving five physical DoF.

These remaining DoF are two charged Higgs bosons (H$^{+/0}$), a CP-odd neutral Higgs boson, A$^0$, and 2 CP-even neutral higgs bosons, h$^0$ and H$^0$.

It is a general prediction of supersymmetric models to expand physical Higgs sectors.
SUSY Higgs Mechanism

After fixing $v_1^2 + v_2^2$ such that W boson gets its correct mass, the Higgs sector is then described by two additional parameters. The usual choice is

$$\tan \beta \equiv \frac{v_2}{v_1}$$

And $M_A$, the mass of the pseudoscalar Higgs boson. Once these two parameters are given, the masses of remaining Higgs bosons can be calculated in terms of $M_A$ and $\tan \beta$. 
The $\mu$ Parameter

The $\mu$ parameters in MSSM is a concern, because this cannot be set 0 since there won’t be EWSB. The mass of $Z$ boson can be written in terms of the radiatively corrected neutral Higgs boson masses and $\mu$;

$$M_Z^2 = 2 \left[ \frac{M_h^2 - M_H^2}{\tan^2 \beta - 1} \tan^2 \beta \right] - 2 \mu^2$$

This requires a sophisticated cancellation between Higgs masses and $\mu$. This cancellation is unattractive for SUSY because this kind of cancellation is exactly what SUSY theories want to avoid.
The Higgs Masses

The neutral Higgs masses are found by diagonalizing the 2x2 Higgs mass matrix. By convention, \( h \) is taken to be the lighter of the neutral Higgs.

At the tree level the neutral Higgs particle masses are:

\[
M_{h,H}^2 = \frac{1}{2} \left\{ M_A^2 + M_Z^2 \mp \sqrt{(M_A^2 + M_Z^2)^2 - 4 M_Z^2 M_A^2 \cos^2 2 \beta} \right\}
\]

The pseudoscalar Higgs particle mass is:

\[
M_A^2 = \frac{2|\mu B|}{\sin 2 \beta}
\]

Charged scalar Higgs particle masses are:

\[
M_{H^\pm}^2 = M_W^2 + M_A^2
\]
Relative Size of SUSY Higgs Masses

The most important predictions from the masses given in the previous page is the relative magnitude of Higgs masses

\[
M_{H^\pm} > M_W \\
M_{H^0} > M_Z \\
M_{h^0} < M_A \\
M_{h^0} < M_Z |\cos 2\beta|
\]

However, the loop corrections to these relationship are large. For instance, Mh receives corrections from t-quark and t-squarks, getting the correction of size \( \sim G_F M_t^4 \).
Loop Corrections to Higgs Masses

The neutral Higgs boson masses become

\[ M_{h,H}^2 = \frac{1}{2} \left\{ M_A^2 + M_Z^2 \right\} \]

\[ \pm \sqrt{ \left( M_A^2 + M_Z^2 \right) \cos 2\beta + \left( h \varepsilon_h \right)^2 + \left( M_A^2 + M_Z^2 \right)^2 \sin^2 2\beta } \]

Where \( \varepsilon_h \) is the one-loop corrections

\[ \varepsilon_h \equiv \frac{3 G_F}{\sqrt{2} \pi^2} M_t^4 \log \left( 1 + \frac{m}{M_t^4} \right) \]

\( M_h \) has upper limit for \( \tan \beta > 1 \).

\[ M_h^2 = M_Z^2 \cos^2 2\beta + \varepsilon_h \]
For given values of $\tan\beta$ and the squark masses, there is an upper bound on the lightest higgs mass at around 110 GeV for a small mixing and 130 GeV for large mixing.

$M_h$ plateaus with $M_A > 300$ GeV
Suggested Reading
