PHYS 5326 – Lecture #25

Monday, Apr. 28, 2003
Dr. Jae Yu

• SUSY and EW Symmetry Breaking
• SUSY Higgs properties
• Squark & Slepton Masses
• Chargino and Neutralino Sectors
• Coupling Constants
• SUSY GUT (SUGRA)
• SUSY Higgs production and decay
Announcement

• Semester project presentation
  – 1:00 – 4:00pm, Wednesday, May 7 in room 200
  – 30 minutes each + 10 minute questions
  – Send me slides by noon, Wednesday, May 7
  – The slides will be made as UTA-HEP notes, thus we need to make the presentations electronic
  – Order of presentation: SH, VK, BS, FJ

• Project reports due at the presentation
  – Must be electronic as well so that they can be made UTA-HEP notes

• Will not have a class next Monday, May 5
SUSY Symmetry Breaking

• The SUSY theory so far contains all SM particles but symmetry is unbroken and particles are massless
• General assumption is that SUSY breaking occurs at very high scale, such as Planck Scale
• The usual approach is the assumption that MSSM is an effective low energy theory
• The SUSY breaking is implemented by by including explicit “soft” mass terms for
  – scalar member of chiral multiplets
  – gaugino member of the vector super-multiplets
• The dimension of soft operators in $L$ must be 3 or less:
  – Mass terms, bi-linear mixing terms (B terms), tri-linear scalar mixing terms (A terms)
• The origins of these SUSY breaking terms are left unspecified…
SUSY Breaking Lagrangian

- The complete set of soft SUSY breaking terms that respects R-parity and SU(3) x SU(2) L x U(1) Y for the first generation is given by the Lagrangian:

\[- \mathcal{L}_{\text{soft}} = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - B \mu \varepsilon_{ij} (H_i^i H_j^j + \text{h.c.}) + M_Q \left( \bar{u}_L u_L + \bar{d}_L d_L \right) + \frac{1}{2} \varepsilon_{ij} \left[ \frac{M_d}{\cos \beta} A_d H_i^i \tilde{Q}^j d_R + \frac{M_e}{\cos \beta} A_e H_i^i \tilde{L}^j e_R + \text{h.c.} \right] + M_u u_R u_R + M_d d_R d_R + M_L e_L e_L + \nu_L \nu_L + M_e e_R e_R + \frac{1}{2} \left[ M_3 g g + M_2 w_i w_i + M_1 b b \right] \frac{g}{\sqrt{2} M_W} \varepsilon_{ij} \left[ \frac{M_d}{\cos \beta} A_d H_i^i \tilde{Q}^j d_R + \frac{M_e}{\cos \beta} A_e H_i^i \tilde{L}^j e_R + \text{h.c.} \right] \]

- This Lagrangian has arbitrary masses for scalars (m_1, m_2, \tilde{M}_Q, \tilde{M}_u, \tilde{M}_d, \tilde{M}_L) and gauginos (M_1, M_2, M_3)
SUSY Breaking Lagrangian Properties

- The mass terms in $L$ breaks the mass degeneracy between particle and their super partners
- The tri-linear $A$-terms defined with explicit factor of mass that affects particles of the third generation
- When $A$ terms are non-zero the scalar partners of the left and right-handed fermions can mix when the Higgs bosons get VEV
- The $B$-term (bi-linear) mixes scalar components of 2 Higgs doublets
- Adding all of the mass and mixing terms to $L$ is allowed by gauge symmetries
- $L_{\text{soft}}$ breaks SUSY but at the expense of more than 50 additional parameters
- Since the gauge interactions in SUSY are fixed, SUSY can still preserve its predictive power
Soft Supersymmetry Breaking

The simplest way to break SUSY is to add all possible soft (scale \( \sim M_W \)) supersymmetry breaking masses for each doublet, along with arbitrary mixing terms, keeping quadratic divergences under control.

The scalar potential involving Higgs becomes

\[
V_H = \left( |\mu|^2 + m_1^2 \right) |\Phi_1|^2 + \left( |\mu|^2 + m_2^2 \right) |\Phi_2|^2 - \mu B \varepsilon_{ij} (\Phi_i^i + \Phi_j^j + h.c)
\]

\[
+ \frac{g^2 + g'^2}{8} \left( |\Phi_1|^2 - |\Phi_2|^2 \right)^2 + \frac{g^2}{2} |\Phi_1^* \cdot \Phi_2|^2
\]

The quartic terms are fixed in terms of gauge couplings therefore are not free parameters.
Higgs Potential of the SUSY

The Higgs potential in SUSY can be interpreted as to be dependent on three independent combinations of parameters

\[ |\mu|^2 + m_1^2 ; \quad |\mu|^2 + m_2^2 ; \quad \mu B \]

Where B is a new mass parameter.
If \( \mu B \) is 0, all terms in the potential are positive, making the minimum, \( <V> = 0 \), back to \( <\Phi_1^0> = <\Phi_2^0> = 0 \).
Thus, all three parameters above should not be zero to break EW symmetry.
SUSY EW Breaking

Symmetry is broken when the neutral components of the Higgs doublets get vacuum expectation values:

\[ \langle \Phi_1 \rangle \equiv v_1; \quad \langle \Phi_2 \rangle \equiv v_2 \]

The values of \( v_1 \) and \( v_2 \) can be made positive, by redefining Higgs fields.

When the EW symmetry is broken, the W gauge boson gets a mass which is fixed by \( v_1 \) and \( v_2 \).

\[ M_W^2 = \frac{g}{2} \left( v_1^2 + v_2^2 \right) \]
SUSY Higgs Mechanism

After fixing $v_1^2 + v_2^2$ such that $W$ boson gets its correct mass, the Higgs sector is then described by two additional parameters. The usual choice is

$$\tan \beta \equiv \frac{v_2}{v_1}$$

And $M_A$, the mass of the pseudoscalar Higgs boson.

Once these two parameters are given, the masses of remaining Higgs bosons can be calculated in terms of $M_A$ and $\tan \beta$. 
The $\mu$ Parameter

The $\mu$ parameters in MSSM is a concern, because this cannot be set 0 since there won’t be EWSB. The mass of Z boson can be written in terms of the radiatively corrected neutral Higgs boson masses and $\mu$;

$$M_Z^2 = 2 \left[ \frac{M_h^2 - M_H^2}{\tan^2 \beta} \tan^2 \beta - 1 \right] - 2 \mu^2$$

This requires a sophisticated cancellation between Higgs masses and $\mu$. This cancellation is unattractive for SUSY because this kind of cancellation is exactly what SUSY theories want to avoid.
The Higgs Masses

The neutral Higgs masses are found by diagonalizing the 2x2 Higgs mass matrix. By convention, h is taken to be the lighter of the neutral Higgs.

At the tree level the neutral Higgs particle masses are:

\[ M_{h, H}^2 = \frac{1}{2} \left\{ M_A^2 + M_Z^2 \mp \sqrt{(M_A^2 + M_Z^2)^2 - 4M_Z^2M_A^2\cos^2 2\beta} \right\} \]

The pseudoscalar Higgs particle mass is:

\[ M_A^2 = \frac{2|\mu B|}{\sin 2\beta} \]

Charged scalar Higgs particle masses are:

\[ M_{H^\pm}^2 = M_W^2 + M_A^2 \]
Relative Size of SUSY Higgs Masses

The most important predictions from the masses given in the previous page is the relative magnitude of Higgs masses

\[ M_{H^\pm} > M_W \]
\[ M_{H^0} > M_Z \]
\[ M_{h^0} < M_A \]
\[ M_{h^0} < M_Z |\cos 2\beta| \]

However, the loop corrections to these relationship are large. For instance, \( M_h \) receives corrections from t-quark and t-squarks, getting the correction of size \( \sim G_F M_t^4 \)
Lightest Higgs Mass vs $M_A$

$M_S = 1 \text{ TeV}$

$\tan \beta = 30$

$\tan \beta = 1.5$

$M_{\text{squark}} = 1 \text{ TeV}$

Assume $A_t = \mu = 0$

No mixing
Maximum Higgs Mass

For large value of $\tan\beta$, $M_h \sim 110\text{GeV}$
Different approach can bring this value up to $\sim 130\text{GeV}$
Maximum Higgs Mass

Minimal SUSY model predicts a neutral Higgs with a mass less than 130GeV

• More complicated SUSY models bring different picture on the mass.
• However, the requirement of Higgs self-coupling remain perturbative gives an upper bound on the lightest SUSY Higgs mass at around 150~175 GeV in all models

What is the physical implication of do not observe higgs at this mass range?

There must be a new physics between the weak (~1TeV) and the Planck scale (~$10^{16}$TeV) which causes the Higgs couplings non-perturbative!!
SUSY Higgs Boson Couplings to Fermions

• The Higgs coupling to fermions dictated by the gauge invariance of the super-potential. At lowest order, it is completely specified by $M_A$ and $\tan\beta$.

• Requiring fermions have their observed masses fixes the couplings in the super-potential:

$$\lambda_D = \frac{gM_d}{\sqrt{2}M_W \cos \beta}; \quad \lambda_U = \frac{gM_u}{\sqrt{2}M_W \sin \beta}; \quad \lambda_L = \frac{gM_l}{\sqrt{2}M_W \cos \beta}$$

Where $g$ is the $SU(2)_L$ gauge coupling, $g^2 = 4\sqrt{2}G_fm^2$

$L$ that contains couplings can be written, in terms of SM couplings, $C_{ffx}$:

$$L = -\frac{gm_i}{2M_W} \left[ C_{ffh} \bar{f}_i f_i h + C_{ffH} \bar{f}_i f_i H + C_{ffA} \bar{f}_i f_i A \right]$$
SUSY Higgs Boson Couplings to Fermions

<table>
<thead>
<tr>
<th>$f$</th>
<th>$C_{ffH}$</th>
<th>$C_{ffH}$</th>
<th>$C_{ffA}$</th>
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<tbody>
<tr>
<td>$u$</td>
<td>$\frac{\cos \alpha}{\sin \beta}$</td>
<td>$\frac{\sin \alpha}{\sin \beta}$</td>
<td>$\cot \beta$</td>
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<tr>
<td>$d$</td>
<td>$-\frac{\sin \alpha}{\cos \beta}$</td>
<td>$\frac{\cos \alpha}{\cos \beta}$</td>
<td>$\tan \beta$</td>
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For SM $C_{ffH} = 1$

MSSM approaches to SM at large $M_A$
MSSM Higgs Couplings

- As $M_A$ becomes large
- $M_{H^+/\pm}$ and $M_{H_0}$ get large too
- Only lightest higgs stays within the spectrum
- The couplings of the lighter Higgs boson to fermions and gauge bosons take on their SM values
- Thus, at large $M_A$ limit ($M_A > 300\,\text{GeV}$), it is difficult to distinguish MSSM from SM
Higgs width depends on the value of tan\(\beta\). 
\(M_h \sim 110\text{GeV}\)
Lightest higgs width is 10-100\text{MeV} while the heavier ones range 0.1-1 \text{GeV}.
Considerably smaller than SM width (a few \text{GeV})

Confined to this region..
Squark and Slepton Masses

If soft SUSY breaking occurs at the scale much larger than $M_Z$, $M_T$, or $A_T$, all soft masses are approximately equal and there will be 12 degenerate squarks.

If the scale is at EWSB, mixing effects become important. For large mixing, one of the stop squarks become lightest in this sector.

\[ M \equiv M_Q = M_u \]
Chargino Sector

• There are two charge 1, spin-1/2 Majorana fermions, winos, and higgsinos.

• The physical mass states, charginos, are linear combinations formed by diagonalizing the mass matrix.

• The chargino matrix is:

\[
M_{\chi^\pm} = \begin{pmatrix}
M_2 & \sqrt{2}M_W \sin \beta \\
\sqrt{2}M_W \cos \beta & -\mu
\end{pmatrix}
\]

Mass eigenstates are:

\[
M_{\chi_{1,2}^\pm} = \frac{1}{2} \left\{ M_2^2 + 2M_W^2 + \mu^2 \pm \left[ (M_2^2 - \mu^2) + 4M_W^4 \cos^2 2\beta \right]^{1/2} \right\}
\]

By convention \(\chi_1\) are the lightest charginos.
Neutralino Sector

- In neutral fermion sector, binos and winos can mix with higgsinos.
- The physical mass states, neutralinos, are linear combinations formed by diagonalizing the mass matrix.
- The neutralino matrix is:

\[
M_\chi^0 = \begin{pmatrix}
M_1 & 0 & -M_Z \cos \beta \sin \theta_W & M_Z \sin \beta \sin \theta_W \\
0 & M_2 & M_Z \cos \beta \cos \theta_W & -M_Z \sin \beta \cos \theta_W \\
-M_Z \cos \beta \sin \theta_W & M_Z \cos \beta \sin \theta_W & 0 & \mu \\
M_Z \sin \beta \sin \theta_W & -M_Z \sin \beta \cos \theta_W & \mu & 0
\end{pmatrix}
\]

\(\theta_W\) is the EW mixing angle. The index \(i\) runs 1-4. The lightest neutralino is usually assumed to be LSP.
Coupling Constants

In both SM and SUSY, coupling strength varies as a function of energy scale. SM, however, the couplings never merge while SUSY it does at around $10^{16}$ GeV.

Thus, SUSY theories can naturally be incorporated into GUT.
SUSY GUT

- Since the coupling constants in SUSY theories unifies at a higher energy scale, the SUSY GUT model is widely accepted.
- In SUSY GUT model, the entire SUSY sectors are described by 5 parameters:

1. A common scalar mass, $m_0$.
2. A common gaugino mass, $m_{1/2}$.
3. A common tri-linear coupling, $A_0$.
4. A Higgs mass parameter, $\mu$.
5. A Higgs mixing parameter, $B$.

This set of assumptions is often called “Superstring inspired SUSY GUT” or SUGRA.
SUSY Higgs Branching Ratios

\( \tan \beta = 2 \), \( A_t - M_h - 1 \text{ TeV, } \mu = 100 \text{ GeV} \)

\( \tan \beta = 30 \), \( A_t - M_h - 1 \text{ TeV, } \mu = 100 \text{ GeV} \)

The branching ratio is very sensitive to \( \tan \beta \).

Invisible mode
Squark and gluino production

Solid line is pp-bar to gluino pairs, dot-dashed is squark pairs, dotted is squark and excited squark, and dashed is squark and gluino.